International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 5 Issue 1, November-December 2020 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

A Study on Power Mean Labeling of the Graphs and Vertex Odd Power Mean Labeling of Graphs

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ABSTRACT

This paper we discuss with power mean labeling of graph and Vertex Odd Power Mean Labeling of Graphs.

A graph $G = (V, E)$ is referred as Power Mean graph with (p, q), if it is feasible to label the vertices $x \epsilon V$ with different elements $f(x)$ from 1, 2, 3, ..., q + 1 in such a way that when each edge $e = uv$ is labeled with

$$\mathrm{f(e = uv)} = \left\lfloor \left(\mathrm{f(u)}^{\mathrm{f(v)}} \mathrm{f(v)}^{\mathrm{f(u)}}\right)^{\frac{1}{\mathrm{f(u)+f(v)}}} \right\rfloor$$

In this paper we define Vertex Odd Power Mean labeling and investigate the same for some graphs.

We define Vertex Odd Power Mean labeling for the graph G(V, E) with p vertices and q edges, if it is feasible to label the vertices $x \epsilon V$ with different labelings f(x) from {1, 3, 5, ..., $2q$ – 1} in such a way that when each edge $e = uv$ is labeled with

$$\mathrm{f(e = uv)} = \left\lfloor \left(\mathrm{f(u)}^{\mathrm{f(v)}} \mathrm{f(v)}^{\mathrm{f(u)}}\right)^{\frac{1}{\mathrm{f(u)+f(v)}}} \right\rfloor$$

$$\text{(or) } \mathrm{f(e = uv)} = \left\lceil \left(\mathrm{f(u)}^{\mathrm{f(v)}} \mathrm{f(v)}^{\mathrm{f(u)}}\right)^{\frac{1}{\mathrm{f(u)+f(v)}}} \right\rceil$$

and the edge labeling are distinct. The graph which admits the Vertex Odd Power Mean labeling, is called Vertex Odd Power Mean graph.

***KEYWORD:** Mean labeling of graph, power mean labeling of graph and Vertex Odd Power Mean Labeling of Graphs*

How to cite this paper: B. Kavitha | Dr. C. Vimala "A Study on Power Mean Labeling of the Graphs and Vertex Odd Power Mean Labeling of Graphs" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-5 | Issue-1, December 2020, pp.1208-1211, URL: www.ijtsrd.com/papers/ijtsrd38116.pdf

IJTSRD38116

INTRODUCTION

A graph G with p vertices and q edges is called a *mean graph* if there is an injective function f from the vertices of G to $\{0,1,\ldots..q\}$ such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even, and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then the resulting edge labels are distinct.

A graph $G = (V,E)$ is referred as Power Mean graph with (p,q), if it is feasible to label the vertices $x \epsilon V$ with different elements $f(x)$ from 1, 2, 3, ..., q + 1 in such a way that when each edge

$e = uv$ is labeled with

$$f(e = uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

In this chapter we have extended the Power Mean labeling for the connected graph $P_m \Theta\ S_3$ and $P_n + S_4$ and it is presented.

Theorem 1.1.

The connected graph $P_n \Theta S_3$ is a Power Mean graph.

Proof

The following graph is $P_n \Theta\ S_3$ with 4n vertices and 4n - 1 edges.

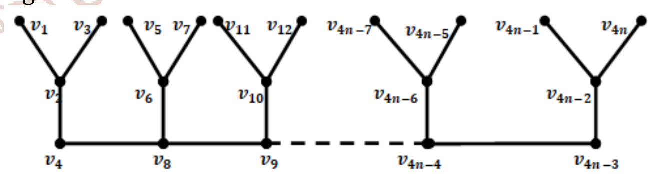

Figure 1.1: Power Mean Labeling of the Graph $P_n \Theta\ S_3$ for odd ‘n’

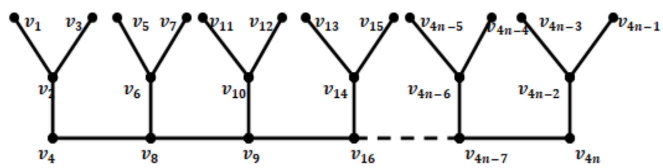

Figure 1.2: Power Mean Labeling of the Graph $P_n \Theta S_3$ for even ‘n’

To find Power Mean Labeling, we define f: V(G) → {1, 2, ..., q} by

$f(v_i) = i$, for $1 \le i \le 4n$.

Hence the edges are labeled as per the definition,

E (G) → {1, 2, ..., q+1} by

$$f(e_i = u_i v_i) = \left\lfloor \left(f(u_i)^{f(v_i)} f(v_i)^{f(v_i)}\right)^{\frac{1}{f(u_i)+f(v_i)}} \right\rfloor$$

or

$$f(e_i = u_i v_i) = \left\lceil \left(f(u_i)^{f(v_i)} f(v_i)^{f(v_i)}\right)^{\frac{1}{f(u_i)+f(v_i)}} \right\rceil$$

And the edge labelings are distinct. Since the graph admits Power Mean labeling, the graph $P_m \Theta$ S_3 is Power Mean graph.

Example 1.1. The connected graph $P_3 \Theta S_3$ is a Power Mean graph.

It has 12 vertices and 11 edges. The graph is labeled as per figure 4.1 and is given below:

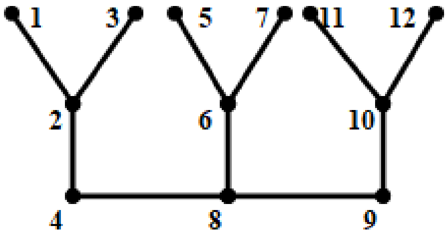

Figure 1.3- Power Mean graph $P_3 \Theta S_3$

Example 1.2. The connected graph $P_4 \Theta S_3$ is a power mean graph.

It has 16 vertices and 15 edges. The graph is labeled as per figure 4.2 and is given below:

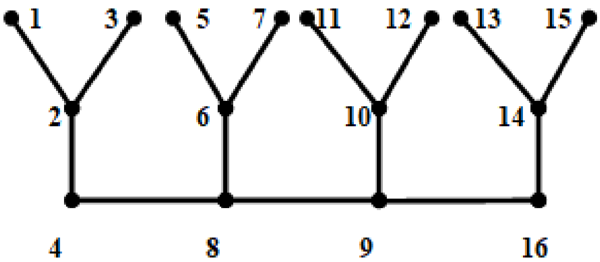

Figure 1.4 -Power Mean graph $P_4 \Theta S_3$

Theorem 1.2.
The connected graph $P_n + S_4$ is a power mean graph.

Proof
The following graph is $P_n + S_4$ with 5n vertices and 5n-1 edges.

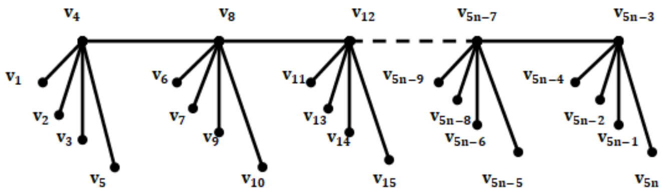

Figure 1.5: Power Mean Labeling of the Graph $P_n + S_4$

To find Power Mean labeling we define V(G) → {1, 2, ..., q} for $P_n + S_4$ as
$f(v_i) = i$, for $1 \le i \le 5n$.

The edges are labeled as per the definition,
E(G) → {1, 2, ..., q} by

$$f(e_i = u_i v_i) = \left\lfloor \left(f(u_i)^{f(v_i)} f(v_i)^{f(v_i)}\right)^{\frac{1}{f(u_i)+f(v_i)}} \right\rfloor$$

Or

$$f(e_i = u_i v_i) = \left\lceil \left(f(u_i)^{f(v_i)} f(v_i)^{f(v_i)}\right)^{\frac{1}{f(u_i)+f(v_i)}} \right\rceil$$

Therefore, the graph holds the definition of Power Mean Labeling and the edge labelings are distinct. Hence the graph $P_n + S_4$ is Power Mean graph.

Example 1.3
The connected graph $P_6 + S_4$ is a Power Mean graph.

This graph $P_6 + S_4$ has 30 vertices and 29 edges. The graph is labeled as per figure 4.5 and is given below:

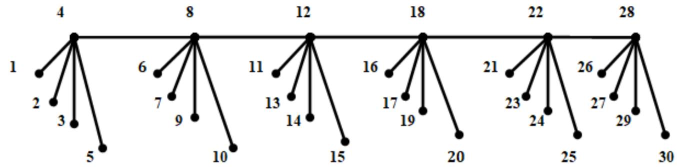

Figure 1.6- Power Mean graph $P_6 + S_4$

Power Mean Graph Labeling
Agraph $G = (V, E)$ is referred as Power Mean graph with (p, q), if it is feasible to label the vertices $x \epsilon V$ with different elements $f(x)$ from 1, 2, 3, ..., $q + 1$ in such a way that when each edge $e = uv$ is labeled with

$$f(e = uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor \text{ (or)}$$

$$f(e = uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

Finding out what has been done for any particular kind of labeling and keeping up with new discoveries, in this thesis we have studied Power Mean labeling and extended it for the graph $P_m + S_n$. Based on the interest on our topic we have also developed another labeling namely 'Odd Vertex Power Mean graph'.

Vertex Odd Power Mean labeling
Vertex Odd Power Mean labeling and investigate the same for some graphs.

We define *Vertex Odd Power Mean labeling* for the graph G(V, E) with p vertices and q edges, if it is feasible to label the vertices $x \epsilon V$ with different labelings $f(x)$ from {1, 3, 5, ..., $2q$ − 1} in such a way that when each edge $e = uv$ is labeled with

$$f(e = uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

$$\text{(or) } f(e = uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

and the edge labelings are distinct. The graph which admits the Vertex Odd Power Mean labeling,

is called *Vertex Odd Power Mean* graph.

Theorem 1.3: The path P_n is vertex odd power mean graph.

Proof: We have the path P$n$of length n.

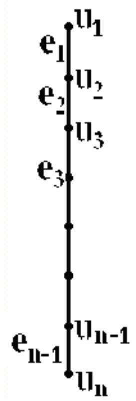

Figure 1.7: Vertex Odd Power Mean labeling of the pathP$_n$

Define a function f: (Pn) → {1,3,...,2q - 1} by

f(x) = v$_i$ = 2i-1 ; 1 ≤ i ≤ n.

The edges are labeled according to the definition. The edge labelings are distinct. Hence f is *vertex odd power mean* labeling. Hence the path Pn is vertex odd Power mean graph.

Example 1.4: A vertex odd power mean labeling of P_7 is given in Figure 1.2.

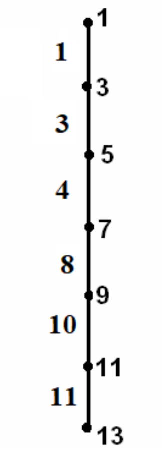

Figure 1.8: Vertex Odd Power Mean labeling of the path P_7

Theorem 1.4: The circuit C_n is vertex odd power mean graph.

Proof: We have the circuit$Cn$of length n.

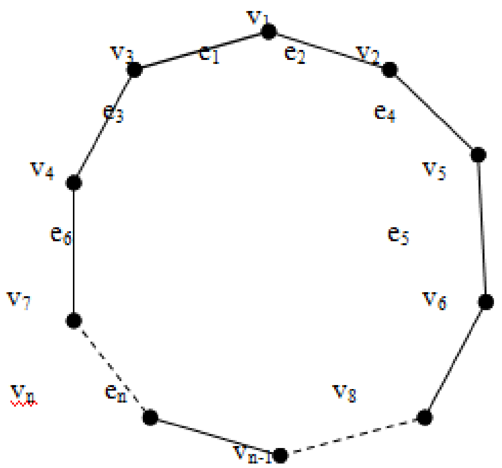

Figure 1.9: Vertex Odd Power Mean labeling of the Cycle C_n

Define a function f: (Cn) → {1,3,...,2q - 1} by

f(x) = v$_i$ = 2i-1 ; 1 ≤ i ≤ n.

The edges are labeled according to thedefinition. The edge labelings are distinct. Hence f is *vertex odd power mean* labeling. Hence the circuit Cn is vertex odd Power mean graph.

Example 1.5: A vertex odd power mean labeling of C_6 is given in Figure1.3.

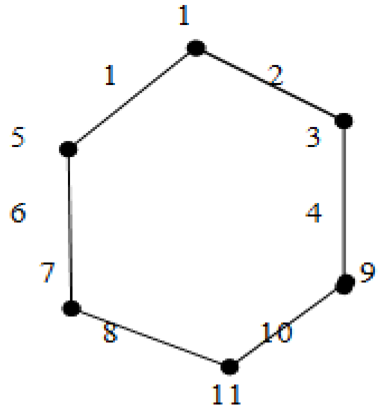

Figure 1.10: Vertex Odd Power Mean labeling of the Cycle C_6

Theorem 1.5: The star graph S_n is Vertex Odd Power Mean graph.

Proof: The star graph S is having one vertex in common. There are n vertices and (n-1) edges.

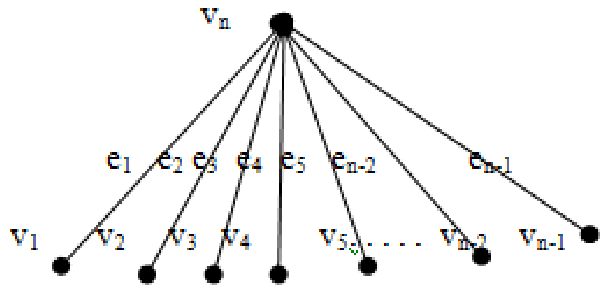

Figure 1.11: Vertex Odd Power Mean labeling of the star S_n

Define a function f: (Cn) → {1, 3, ...,2q - 1} by

f(x) = v$_i$ = 2i-1 ; 1 ≤ i ≤ n.

Hence the edge labeling will be f(e = uv) = e_1 = 1, and f(e_i) = 2i, i = 2, 3, 4, ..., n-1

The edges are labeled according to the definition 1. The edge labelings are distinct. Hence f is vertex odd power mean labeling. Hence the circuit Cn is vertex odd Power mean graph.

Example 1.6: A vertex odd power mean labeling of S_6 is given in Figure 1.4.

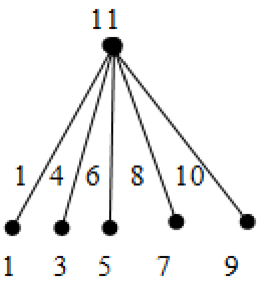

Figure 1.12: Vertex Odd Power Mean labeling of the Star S_n

Theorem 1.6: The connected graph $P_m + C_n$ is vertex odd power mean graph.

Proof: The connected graph $P_m + C_n$ has m vertices and n edges in common. There are n vertices and n edges.

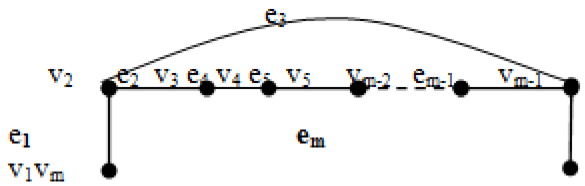

Figure 1.13: Vertex Odd Power Mean labeling of the graph Pm + C_n

Define a function $f: (Cn) \rightarrow \{1,3,\ldots,2q - 1\}$ by
$f(x) = v_i = 2i - 1$; $1 \leq i \leq n$.

The edges are labeled according to the definition 1. The edge labelings are distinct. Hence f is vertex odd power mean labeling. Hence the circuit Cn is vertex odd Power mean graph.

Example 1.7: A vertex odd power mean labeling of the graph Pm + C_n is given in Figure 1.5.

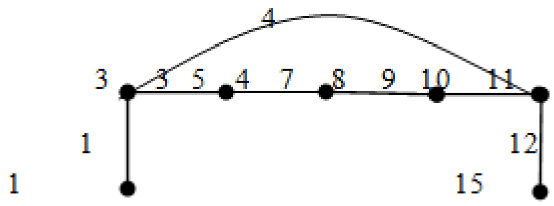

Figure 1.14: Vertex Odd Power Mean labeling of the graph $P_m + C_5$

Example 1.8

The connected graph $P_7 + C_5$is a vertex odd power mean graph.

It has 7 vertices and 7 edges. The graph is labeled as per figure 5.7 and is given below:

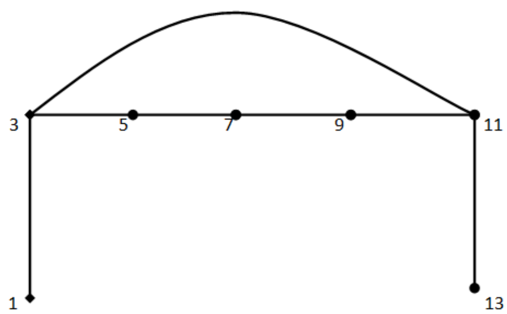

Figure 1.15: Vertex Odd Power Mean labeling of the graph $P_7 + C_5$

Example 1.9

The connected graph $P_8 + C_6$is a vertex odd power mean graph.

This graph $P_8 + C_6$has 8 vertices and 8 edges. The graph is labeled as per figure 5.6 and is given below:

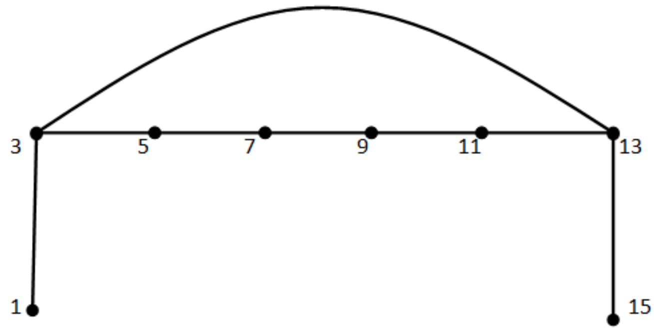

Figure1.16: Vertex Odd Power Mean labeling of the graph $P_8 + C_6$

In this chapter, we have investigated the Power Mean labeling for the graphs $P_m \Theta S_3$ and $P_n + S_4$. And alsoVertex Odd Power mean graphof $P_m + C_n$.

Conclusion:

In this thesis we extended the power mean labeling for the graphs $P_m \Theta S_3$ and $P_n + S_4$. Vertex odd power mean labeling is found for $P_m + C_n$.

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